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DETECTION OF WIND DRIVEN INERTIAL OSCILLATIONS IN THE EASTERN IRISH SEA

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Abstract. Inertial oscillations are essentially anticyclonic in the northern hemisphere. If the tidal ellipses are cyclonic, the current velocity spectrum can be resolved into its clockwise and anticlockwise components. The presence of the inertial currents is marked by a peak in the clockwise spectrum corresponding to the inertial frequency 'f'. In the present study, as no such peak was observed in the spectrum, it was concluded that no inertial oscillations were present during the period of observations. This was expected on theoretical grounds in addition to the lack of a suitable wind stress needed to initiate such currents. In the relatively shallow waters of the Eastern Irish sea bottom friction is likely to be felt over an appreciable fraction of the total depth and so inertial oscillations are unlikely to exist.

Introduction

Horizontal inertial currents turning in an anticyclonic direction with a period of 12/Sin ϕ hr (where ϕ is the latitude) may be found at all depths in the oceans, enclosed seas and great lakes. These oscillations are essentially transient phenomena and they occur in thin layers with a relatively reduced horizontal extent. For the surface waters of the oceans the wind stress is the principal mechanism for their generation. The transmission of the movement into deeper layers may be accomplished by one of the two possible processes. (1) by turbulence in an unbounded homogenous medium (2) by boundary effects (bottom topography, local variations in wind stress) in a strongly stratified medium.

Any system of currents may be superimposed on inertial currents. For example, a simple translatory motion superimposed on an inertial current would stretch the inertial circle into a series of loops in the direction of the translatory motion. Also frictional effects must gradually reduce the speed of the inertial currents and also reduce the radius of the inertial circle.

The question arises as to the possibility of the existence of wind driven inertial oscillations in the Irish sea. Inertial currents have been observed by Gustafson and Kullenburg[1] in the Baltic sea, which is about twice the size of the Irish sea (both horizontally and vertically). Both spatial scales favour the Baltic as an area more suitable for inertial oscillations, but the differences are not large enough to preclude the existence of such oscillations in the Irish sea.

On the basis of a zero-slope model of wind-driven currents Hunter[2] predicted that over a substantial area of the Irish sea (73%) conditions appear suitable for the generation of inertial oscillations. Out of the remaining 27% most of the area is confined to the west coast of England and Wales.

In the presence of tidal currents, inertial currents can often be difficult to detect, particularly if the inertial period falls close to that of one of the predominant tidal constituents. However, if the tidal ellipses are cyclonic, i.e. anticlockwise in the northern hemisphere, then the inertial current can be resolved by separating the current velocity spectrum into a clockwise and anticlockwise component.[3]

Data Analysis. The data from the two current meter mooring positions A (53° 42'N, 3° 32.5'W) and B (53° 35'N 3° 22'W) shown in Fig. 1 in the Eastern Irish sea was analysed in terms of residual currents. At position A during 1975 (31st July–13th August) three current meters nos 629, 626, 556 were at a distance of 3M, 12.5M and 19M and in 1976 (30th June – 10th July) meter nos 1749, 1750 and 1867 were at the same distance respectively from the sea floor with a total depth of 24M at lowest low water. At position B in 1975 (31st July–13th August) meter no. 406 was 11M and meter no. 236 was 4M above the sea floor in a total depth of 17M at lowest low water.

Typical plot of the progressive vector diagram based on 5 min values for current meter No. 629 is shown in

Fig. 2. The low pass filter $\frac{\alpha^{2}_{12}\cdot\alpha_{14}}{12^{2}\cdot 14}$ was used in the present work for smoothing and decimating the observations from a 5 min to a 1 hr sampling interval. Progressive vector diagrams based on hourly residual values, obtained by the application of $\frac{\alpha^{2}_{24}\cdot\alpha_{25}}{24^{2}\cdot 25}$ to smoothed hourly values were constructed and are shown in Figs. 3, 4, 5.

The Spectrum. The power spectrum curve shows how the variance of a random process is distributed over the different frequencies. The application of Fourier series[4] helps in decomposing the variance or average power of the signal X(t), into harmonics of the fundamental frequency. If we define power as an average energy in the time series i.e.

$$\text{Power} = \frac{1}{N}\sum_{n=0}^{N-1} x_n^{2}$$

it can be shown that $\frac{1}{N}\sum_{n=0}^{N-1} x_n^{2} = \sum_{k=0}^{N-1} \int x_k \int^{2}$

$$\text{where } X_k = \sum_{n=0}^{N-1} x_n \exp\frac{(-j2\pi Kn)}{N}$$

If we define 'Power spectral density' S_k where $(S_k 1/T)$ is the power (average energy) in the Kth frequency interval of width df = 1/T, then the total power in the frequency spectrum = total power in the time series i.e.

$$\sum_{k=0}^{N-1} S_k\cdot\frac{1}{T} = \sum_{k=0}^{N-1}\int x_k \int^{2}$$

or

$$\sum_{k=0}^{N-1} S_k = T\sum_{k=0}^{N-1}\int x_k \int^{2}$$

If we define the power spectrum G_k in terms of positive frequencies only, then,

$$G_k = S_k + S_{-k} = 2S_k$$

or

$$\Sigma G_k = 2T\Sigma \int x_k \int^{2}$$

If we define the kinetic energy = $1/2\, u^2$ where u is velocity then

$$\Sigma G_k = T\Sigma \int x_k \int^{2}$$

where X_k is derived from a time series u_n of velocity samples.

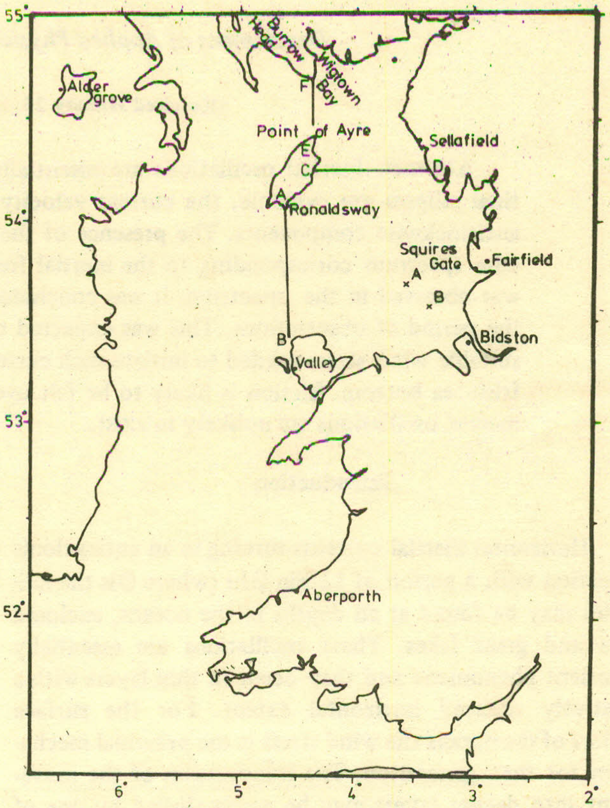

Fig. 1. Map of Irish Sea showing current meter mooring positions A and B in the Eastern Irish Sea.

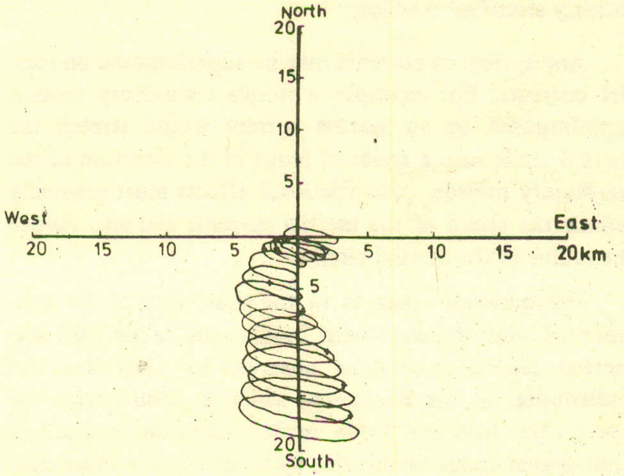

Fig. 2. Progressive vector diagram based on 5 minutes values for meter No. 629; X at 1145 BST each day.

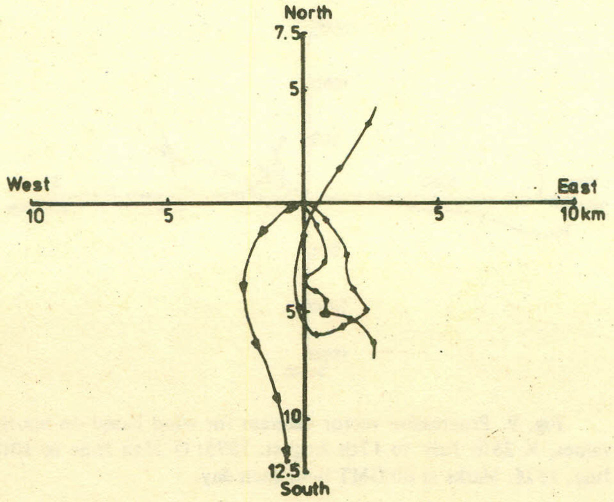

Fig. 3. Progressive vector diagrams based on hourly residual values (Godin's filter) for mooring position A in 1975. X Top, meter 556; O Middle, meter 626; ▶ Bottom, meter 629; Marks at 1147½ BST each day.

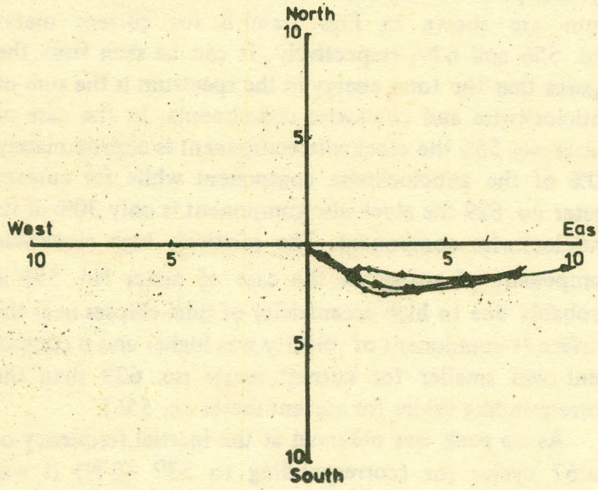

Fig. 4. Progressive vector diagrams based on hourly residual values (Godin's filter) for mooring position A in 1976. X Top, meter 1867; O Middle, meter 1750; ▶ Bottom, meter 1749; Marks at 1142½ BST each day.

Examples of energy spectrum curves of u, v components of velocity for meter no. 556 are shown in Figs. 6a and 6b, respectively. It is clear from the curves that most of the energy was due to M_2 (Lunar Semi-diurnal) and S_2 (Solar semi-diurnal) components of the tidal constituents. Also the comparison of these two figures shows that much of the energy is due to the u component of velocity which is higher than the v component.

Spectral Analysis of the Vector Series. A vector time series can be represented by C=u+jv where C represents the vector velocity and u and v are the scalar components of C along rectangular axes. The angular velocity may be positive (anticlockwise rotation) or negative (clockwise rotation). In the northern hemisphere, inertial currents are essentially in a clockwise direction and if the tidal ellipses are anticlockwise, the spectrum can conveniently be borken into clockwise and anticlockwise components.

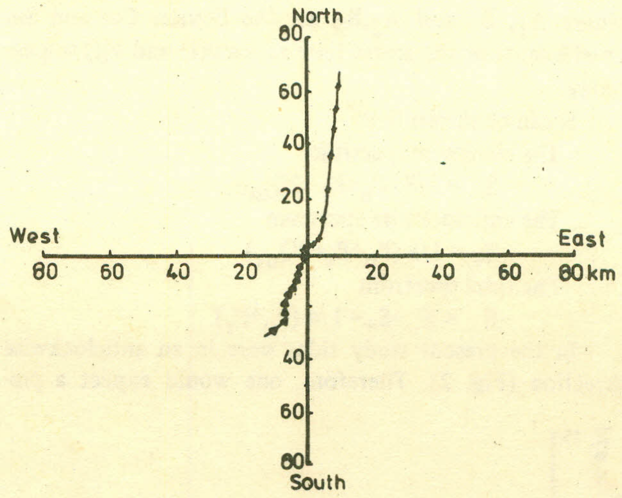

Fig. 5. Progressive vector diagrams based on hourly residual values (Godin's filter) for mooring position B in 1975. X Top, meter 406; ▶ Bottom, meter 236; Marks at 1132½ BST each day.

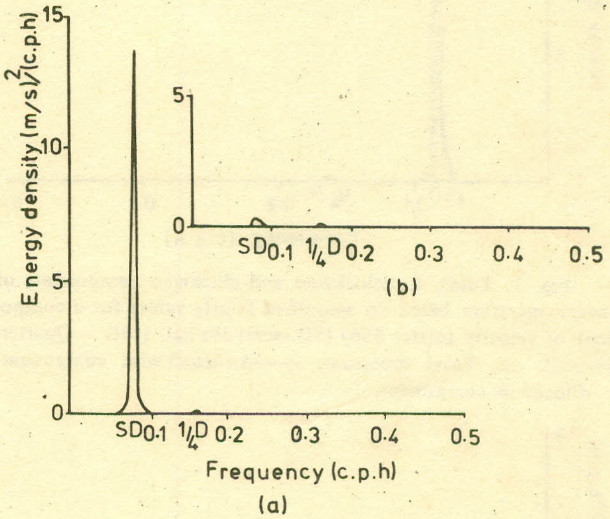

Fig. 6. Energy Spectrum based on hourly tidal values for current meter No. 556 (SD–Semi-diurnal, 1/4D – Quarterdiurnal) (a) u component of velocity; (b) v component of velocity.

If P_u and P_v are the autospectra of the scalar components u and v and P_{uv} and Q_{uv} are the cross and quadrature spectra then[3]

$$P_u = \langle A_1^2 + B_1^2 \rangle$$

$$P_v = \langle A_2^2 + B_2^2 \rangle$$

$$P_{uv} = \langle A_1A_2 + B_1B_2 \rangle$$

$$Q_{uv} = \langle A_1B_2 - A_2B_1 \rangle$$

where A_1, B_1 and A_2,B_2 are the Fourier Cos and Sin co-efficients of the scalar time series u(t) and v(t) respectively.

It can be shown that[3]

The clockwise spectrum

$$S_- = 1/8\,(P_u+P_v-2Q_{uv})$$

The anticlockwise spectrum

$$S_+ = 1/8\,(P_u+P_v+2Q_{uv})$$

The total spectrum

$$S = S_- + S_+ = 1/4\,(P_u+P_v)$$

In the present study tides were in an anticlockwise direction (Fig. 2). Therefore, one would expect a predominant anticlockwise component. If inertial currents are present then there should be a peak in the clockwise spectrum, corresponding to the inertial frequency 'f'. Typical plots of total, clockwise and anticlockwise spectrum are shown in Figs. 7 and 8 for current meter nos. 556 and 629, respectively. It can be seen from the figures that the total energy in the spectrum is the sum of anticlockwise and clockwise components. In the case of meter no. 556 the clockwise component is approximately 70% of the anticlockwise component while for current meter no. 629 the clockwise component is only 30% of its anticlockwise component. The relatively high clockwise component of energy in the case of meter No. 556 is probably due to high eccentricity of tidal ellipses near the surface (v component of velocity was higher and u component was smaller for current meter no. 629 than the corresponding values for current meter no. 556).

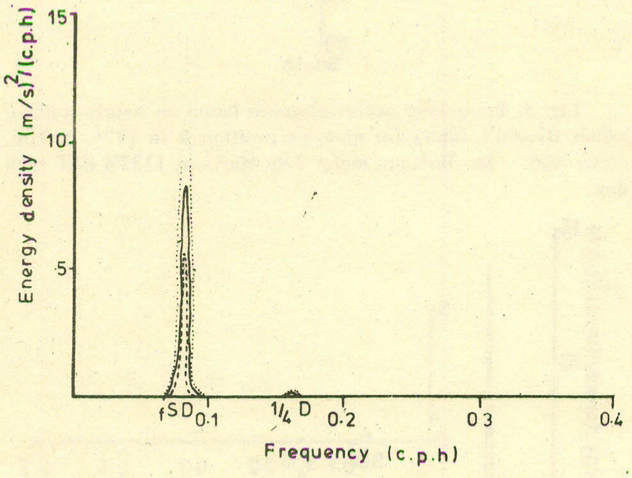

Fig. 7. Total, Anticlockwise and clockwise components of energy spectrum based on smoothed hourly values for u component. of velocity (meter 556) (SD–semi-diurnal; 1/4D – Quarter-diurnal); ..... Total spectrum; ——Anticlockwise component, – –Clockwise component.

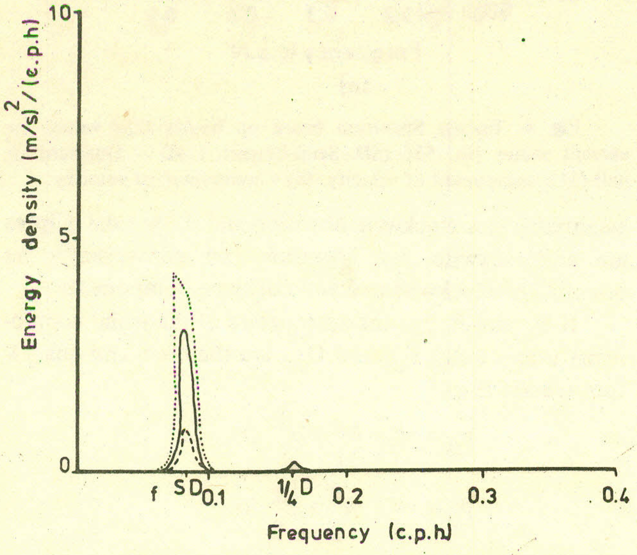

Fig. 8. Total, Anticlockwise and clockwise components of energy spectrum based on smoothed hourly values for u component of velocity (meter 629). (SD – semi-diurnal; 1/4D – Quarter-diurnal); ..... Total spectrum; —— Anticlockwise component; – –Clockwise component.

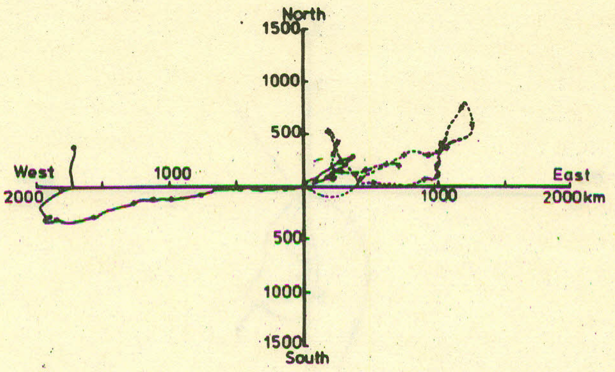

Fig. 9. Progressive vector diagram for wind based on hourly values, X 25th July to 13th August, 1975; O 25th June to 10th July, 1976; Marks at 00 GMT hour each day.

As no peak was observed at the inertial frequency of 0.067 cycles /hr (corresponding to 53° 42'N) it was concluded that no inertial current components were present during the period of observation.

In order to further investigate any possible relation between the observed currents and the wind, progressive vector diagrams based on the average wind from three stations (Squires Gate, Ronaldsway and Valley) for the periods 25th July to 13th August 1975 and 25th June to 10th July 1976 were constructed and are shown in Fig. 9. Comparison of progressive vector diagram of wind with those of residual currents Figs. 3, 4, 5 for the respective years show that the residual currents bore no relation to the wind velocity.

Conclusions

The experimental result that inertial currents were absent from the current meter data was to be expected on theoretical grounds, in addition to the lack of a suitable wind stress needed to initiate such currents. The theoretical assumptions made in the development of the equations describing inertial currents $T_p = 12/\mathrm{Sin}\phi$/hr include the assumption that the horizontal flow is frictionless. In the relatively shallow waters of the Eastern Irish sea bottom friction is likely to be felt over an appreciable fraction of the total depth, and so inertial currents are unlikely to exist. These observations also give support to the Hunters[2] model that inertial oscillations are not present in the shallow area of the west coast of England.

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